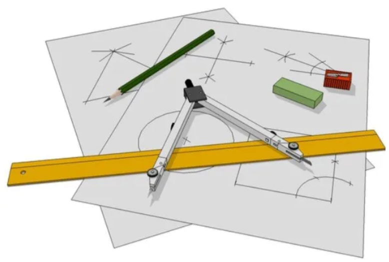
2 Topic: Straightedge and compass constructions

The goal of this project is to learn the ways the Greeks used the unmarked straightedge and collapsible compass to present the only way (according to the Greeks!) to assert that a mathematical object exists — construct it; many of these constructions deepen our understanding of fundamental geometric relationships, and are also much fun.



$\diamondsuit$ **2.1.** (A warm up, please do not include in your report.) Reproduce the three constructions that lead to the proofs of the first three propositions in the first book of *Elements*. Why do you think Euclid started his book with these three propositions?

$\diamondsuit$ **2.2.** Solutions to the following problems should include two steps: (a) an algorithm that lists all the required steps one by one, and (b) a proof that the suggested algorithm actually produces what is required.

1. Construct a bisector for the given proper angle.
2. Construct a perpendicular bisector for the given segment.
3. For a given line and a point on the line constructs the line through the point perpendicular to the line.
4. For a given line and a point not on the line constructs the line through the point perpendicular to the line.

$\diamondsuit$ **2.3.** Given a line segment divide it into n equal parts. *Hint:* Let AB be a given segment. Start with drawing a ray from A that forms an acute angle with AB. Construct n equal segments on this ray (of arbitrary length). Realize how to finish the construction. Prove that your algorithm yields what is required.

$\diamondsuit$ **2.4.** Here is a straightedge and compass "solution" of the problem to trisect an angle. On the two rays of the angle find points A and B, such that $OA = AB$, where O is the vertex. Divide segment AB into three equal parts AC, CD, DB (by Problem 7 it is always possible). Connect O with C and D. Problem is solved! What is wrong with this "solution"? (For extra credit: Can you provide a formal argument that the suggested solution is incorrect?)